CHAPTERWISE QUESTION

MATHEMATICS

SET A

APPLICATIONS OF DERIVATIVES

CLASS - XII **Time : 1½ hrs.**

Marks : 40

SECTION - A **10 × 1 = 10**

1. If the function f(x) = $2x^2 - kx + 7$ is increasing on [1, 2] then k lies in the interval.

   a) $(-\infty, 4)$ b) $(4, \infty)$ c) $(-\infty, 8)$ d) $(8, \infty)$

2. The function f(x) = $\log_e \left(x^3 + \sqrt{x^6+1}\right)$ is of following type

   a) even and increasing b) odd and increasing

   c) odd and decreasing d) even and decreasing

3. The interval on which the function f(x) = sinx – cosx, $0 < x < 2\pi$ is strictly decreasing on

   a) $\left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$ b) $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$ c) $\left(\frac{-\pi}{4}, \frac{5\pi}{4}\right)$ d) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

4. The value of a for which f(x) = a(x + sinx) + a is increasing on R

   a) $a \in (0, \infty)$ b) $a \in (10, \infty)$ c) $a \in (-\infty, 0)$ d) None of these

5. Let x and y be two variable and x > 0, xy = 1, then minimum value of x + y is

   a) 1 b) 2 c) $2\frac{1}{2}$ d) $3\frac{1}{2}$

6. If x is real, the minimum value of $x^2 - 8x + 17$ is

   a) –1 b) 0 c) 1 d) 2

7. The side of an equilateral triangle is increasing at the rate of 2 cm/s. The rate at which area increases when the side is 10 is

   a) 10 cm^2/s b) $\sqrt{3}\ cm^2/s$ c) $10\sqrt{3}\ cm^2/s$ d) $\frac{10}{3}\ cm^2/s$

8. The rate of change of area of a circle with respect to its radius *r* at r = 6 cm is

   a) 10π b) 12π c) 11π d) 8π

For question number 9 -10 two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c) Assertion (A) is true but Reason (R) is false.

d) Assertion (A) is false but Reason (R) is true.

9. Assertion (A) : A function *f* given $f(x) = x^3 - 3x^2 + 4x,\ x \in R$ is strictly increasing on R.

Reason (R) : A real valued function *f*(x) is strictly increasing in R if $f'(x) > 0$ in every interval of R.

10. Assertion (A) : The maximum value of the function sinx + cosx is $\sqrt{2}$

Reason (R) : Sinx $\in$ [-1, 1] for all $x \in R$ and cosx $\in$ [-1, 1] for all $x \in R$

SECTION - B **2 × 2 = 4**

11. Radius of a variable circle is changing at the rate of 5 cm/s. What is the radius of the circle at a time when its area is changing at the rate of 100 cm^2/s?

OR

Find the point on the curve $y = x^2$, where the rate of change of x-coordinate is equal to the rate of change of y-coordinate.

12. Find the maximum and minimum values if any of the function given by $f(x) = -(x-1)^2 + 10$.

SECTION - C **4 × 3 = 12**

13. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of the material will be the least when the depth of the tank is half of its width.

14. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening.

15. The radius *r* of a right circular cylinder is decreasing at the rate of 3 cm/min and its height *h* is increasing at the rate of 2 cm/min. When *r* = 7 cm and *h* = 2 cm, find the rate of change of the volume of cylinder. $\left[\text{Use } \pi = \frac{22}{7}\right]$

16. The length *x* of a rectangle is decreasing at the rate of 5 cm/minute and the width *y* is increasing at the rate of 4 cm/minute. When *x* = 8 cm and *y* = 6 cm, find the rate of change of (a) the perimeter, and (b) the area of the rectangle.

OR

A balloon which always remains spherical has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

SECTION - D **2 × 5 = 10**

17. Find the values of *x* for which the function $f(x) = [x(x-2)]^2$ is an increasing function. Also, find the points on the curve, where the tangent is parallel to the *x*-axis.

18. Find the intervals in which the function *f* given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$, is strictly increasing or strictly decreasing.

OR

Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex coinciding with one extremity of the major axis.

SECTION - E

Case Study

19. An architecture design in an auditorium for a school for its cultural activities is shown below. The floor of the auditorium is rectangular in shape and has a fixed perimeter P.

Based on the above information, answer the following questions.

i) If 'x' and 'y' represents the length and breadth of rectangular region, then write the perimeter by using the relation between x and y. **1**

ii) Find the area (A) of the rectangular region as a function of 'x'. **2**

iii) School manager is interested in maximising the area (A) of floor, for this what should be the value of 'x'?

OR

iii) Find the value of y, for which the area of floor is maximum. **1**

CHAPTERWISE QUESTION

MATHEMATICS

SET B

APPLICATIONS OF DERIVATIVES

CLASS - XII **Time : 1½ hrs.**

Marks : 40

SECTION - A **10 × 1 = 10**

1. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has

   a) two points of local maximum b) two points of local minimum

   c) one maximum and one minimum d) no maximum no minimum

2. The maximum value of sinx.cosx is

   a) 1/4 b) 1/2 c) $\sqrt{2}$ d) $2\sqrt{2}$

3. The function $f(x) = \frac{1+4x^2}{x}, x \neq 0$ is increasing on

   a) $(-\infty, -1) \cup [1, \infty)$ b) $(-\infty, -1/2) \cup [-1/2, \infty]$

   c) $(-\infty, -1/2] \cup [1/2, \infty)$ d) None of these

4. The least value of 'a ' such that the function *f* given by $f(x) = x^2 + ax + 1$ is increasing on [1, 2]

   a) 1/2 b) –1 c) –4 d) –2

5. The function *f* given by $f(x) = x^2 - x + 1$ is

   a) strictly increasing on (-1, 1) b) strictly decreasing on (-1/2, 1)

   c) neither strictly increasing nor strictly decreasing on (-1,1) d) None of these

6. The $f(x) = e^{-x}$ is strictly decreasing function on

   a) R-1 b) R-0 c) R d) None of these

7. A spherical balloon has a variable diameter $\frac{3}{2}(2x+1)$. The rate of change of its volume with respect to *x* is

   a) $\frac{27\pi}{8}(2x+1)^2$ b) $\frac{9}{4}\pi(2x+1)^3$ c) $\frac{9\pi}{16}(2x+1)^3$ d) $\pi(2x+1)^2$

8. If the rate of change of area of the circle is equal to the rate of change of its diameter then its radius is equal to

   a) π units b) $\frac{1}{\pi}$ units c) $\frac{\pi}{2}$ units d) $\frac{2}{\pi}$ units

For question number 9 -10 two statements are given - one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c) Assertion (A) is true but Reason (R) is false.

d) Assertion (A) is false but Reason (R) is true.

9. Assertion (A) : A function given by f(x) = log sinx is strictly increasing on $(0, \pi/2)$

   Reason (R) : The function f(x) = sinx is strictly increasing in $(0, \pi/2)$

10. Assertion (A) : The function f(x) = x - sinx is increasing for all $x \in R$

    Reason (R) : The domain of the function f(x) = x - sinx is R.

SECTION - B **2 × 2 = 4**

11. Find the maximum and minimum values if any of the function given by *f*(x) = sin 2x + 5.

OR

Show that $y = e^x$ has no local maxima or local minima.

12. Find the intervals in which the function *f* given by $f(x) = x^3 - 12x^2 + 36x + 17$ in increasing or decreasing.

SECTION - C **4 × 3 = 12**

13. The volume of a sphere is increasing at the rate of 8 cm^3/s. Find the rate at which its surface area is increasing when the radius of the sphere is 12 cm.

14. A stone is dropped into a quiet lake and waves move in circles at a speed of 5 cm per second. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

OR

Of all the rectangles each of which has perimeter 40 metres, find one which has maximum area. Find the area also.

15. A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12m, find the dimensions of the rectangle that will produce the largest area of the window.

16. Find the intervals in which the function $f(x) = \frac{3}{2}x^4\, x^4 - 4x^3 - 45x^2 + 51$ is

i) strictly increasing ii) strictly decreasing

SECTION - D **2 × 5 = 10**

17. A given quantity of metal is to be cast into a half cylinder with a rectangular base and semicircular ends. Show that in order that total surface area is minimum, the ratio of length of cylinder to the diameter of its semicircular ends is $\pi : (\pi + 2)$.

18. Find the intervals in which the function *f* given by *f*(x) = sin x + cos x, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

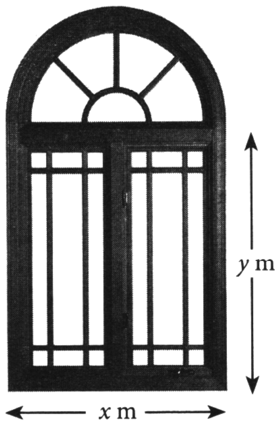
OR

A metal box with a square base and vertical sides is to contain 1024 cm^3. The material for the top and bottom costs ₹ 5 per cm^2 and the material for the sides costs ₹ 2.50 per cm^2. Find the least cost of the box.

SECTION - E

Case Study

19. Rohan, a student of class XII, visited his uncle's flat with his father. He observed that the window of the house is in the form of a rectangle surmounted by a semicircular opening having perimeter 10 m as shown in the figure.



Based on the above information, answer the following questions.

i) If 'x' and 'y' represents the length and breadth of rectangular region, how can be represent the relation between 'x' and 'y'. **1**

ii) Rohan is interested in maximizing the area of the whole window, for this what should be the value of 'x'? **2**

OR

Find the maximum area of the window.

iii) For maximum value of Area (A), what will be the breadth of rectangular part of the windows. **1**